Definitions

Let $r = r_0 + \alpha r_d$ be a ray, and let $0 = \frac{(x-e_x)^2}{e_w^2} + \frac{(y-e_y)^2}{e_h^2} + \frac{(z-e_z)^2}{e_d^2} - 1$ be an ellipsoid surface.

Intersection

$$0 = \frac{(\alpha r_d(x) + r_0(x) - c_x)^2}{e_w^2} + \frac{(\alpha r_d(y) + r_0(y) - c_y)^2}{e_h^2} + \frac{(\alpha r_d(z) + r_0(z) - c_z)^2}{e_d^2} - 1$$

$$= \frac{\alpha^2 r_d(x)^2 + \alpha r_d(x) \cdot 2 \cdot (r_0(x) - c_x) + (r_0(x) - c_x)^2}{e_w^2}$$

$$+ \frac{\alpha^2 r_d(y)^2 + \alpha r_d(y) \cdot 2 \cdot (r_0(y) - c_y) + (r_0(y) - c_y)^2}{e_h^2}$$

$$+ \frac{\alpha^2 r_d(z)^2 + \alpha r_d(z) \cdot 2 \cdot (r_0(z) - c_z) + (r_0(z) - c_z)^2}{e_d^2} - 1$$

$$= \alpha^2 \frac{r_d(x)^2}{e_w^2} + \alpha \frac{r_d(x) \cdot 2 \cdot (r_0(x) - c_x)}{e_w^2} + \frac{(r_0(x) - c_x)^2}{e_w^2}$$

$$+ \alpha^2 \frac{r_d(y)^2}{e_h^2} + \alpha \frac{r_d(y) \cdot 2 \cdot (r_0(y) - c_y)}{e_h^2} + \frac{(r_0(y) - c_y)^2}{e_h^2}$$

$$+ \alpha^2 \frac{r_d(z)^2}{e_d^2} + \alpha \frac{r_d(z) \cdot 2 \cdot (r_0(z) - c_z)}{e_d^2} + \frac{(r_0(z) - c_z)^2}{e_d^2} - 1$$

$$= \alpha^2 \cdot \left( \frac{r_d(x)^2}{e_w^2} + \frac{r_d(y)^2}{e_h^2} + \frac{r_d(z)^2}{e_d^2} \right)$$

$$+ \alpha \left( \frac{r_d(x) \cdot 2 \cdot (r_0(x) - c_x)}{e_w^2} + \frac{r_d(y) \cdot 2 \cdot (r_0(y) - c_y)}{e_h^2} + \frac{r_d(z) \cdot 2 \cdot (r_0(z) - c_z)}{e_d^2} \right)$$

$$+ \frac{(r_0(x) - c_x)^2}{e_w^2} + \frac{(r_0(y) - c_y)^2}{e_h^2} + \frac{(r_0(z) - c_z)^2}{e_d^2} - 1$$

$$0 = ax^2 + bx + c \leftrightarrow 0 = x^2 + \frac{b}{a} \cdot x + \frac{c}{a} = x^2 + px + q, p = \frac{b}{a}, q = \frac{c}{a}$$